July 31. (Extraordinary Meeting.)

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

RESOLVED,—On the recommendation of Council,—That the Treasurer be empowered to sell stock in the 3 per cent. Consols, to the amount of £300, in order to pay Mr. Gill's bill for printing Transactions to March 16, 1843, amount £264 10*s*. 4*d*., and the rent of the Academy House to 31st July, 1843.

RESOLVED,—On the recommendation of Council,—That the Treasurer be empowered to sell such $3\frac{1}{2}$ per cent. stock, being the Cunningham Fund, as shall amount to £50, towards defraying the cost of medals.

Sir William Betham presented to the Academy certain casts from the sculptures on the inside of the tower of Ardmore.

Dr. Lloyd having taken the Chair, the President gave an account of some researches in the Calculus of Probabilities.

Many questions in the mathematical theory of probabilities conduct to approximate expressions of the form

$$p = \frac{2}{\sqrt{\pi}} \int_0^t dt\, e^{-t};$$

that is,

$$p = \theta(t),$$

θ being the characteristic of a certain function which has been tabulated by Encke in a memoir on the Method of Least Squares, translated from the Berlin Ephemeris, in vol. ii. part 7 of Taylor's Scientific Memoirs; p being the probability sought, and t an auxiliary variable.

Sir William Hamilton proposes to treat the equation

$$p = \theta(t)$$

as being in all cases rigorous, by suitably determining the auxiliary variable t, which variable he proposes to call the

argument of probability, because it is the argument with which Encke's Table should be entered, in order to obtain from that Table the value of the probability p. He shows how to improve several of Laplace's approximate expressions for the argument t, and uses in many such questions a transformation of a certain double definite integral, of the form,

$$\frac{4s^{\frac{1}{2}}}{\pi}\int_0^r dr \int_0^\infty du\, e^{-su^2}\, \mathrm{U} \cos(2s^{\frac{1}{2}} ru\, \mathrm{V})$$
$$= \theta(r(1+\nu_1 s^{-1} + \nu_2 s^{-2} + \ldots));$$

in which

$$\mathrm{U} = 1 + a_1 u^2 + a_2 u^4 + \ldots$$
$$\mathrm{V} = 1 + \beta_1 u^2 + \beta_2 u^4 + \ldots$$

while $\nu_1, \nu_2, \ldots$ depend on $a_1, \ldots \beta_1, \ldots$ and on r; thus

$$\nu_1 = \tfrac{1}{2} a_1 - \beta_1 r^2.$$

The function θ has the same form as before, so that if, for sufficiently large values of the quantity s (which represents, in many questions, the number of observations or events to be combined), a probability p can be expressed, exactly or nearly, by the foregoing double definite integral, then the *argument* t, of this probability p, will be expressed nearly by the formula,

$$t = r(1 + \nu_1 s^{-1} + \nu_2 s^{-2}).$$

Numerical examples were given, in which the approximations thus obtained appeared to be very close. For instance, if a common die (supposed to be perfectly fair) be thrown six times, the probability that the sum of the six numbers which turn up in these six throws shall not be less than 18, nor more than 24, is represented rigorously by the integral

$$p = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} dx \frac{\sin 7x}{\sin x}\left(\frac{\sin 6x}{6 \sin x}\right)^6, \text{ or by the fraction } \tfrac{27448}{46656};$$

while the approximate formula deduced by the foregoing method gives 27449 for the numerator of this fraction, or for the product $6^6 p$; the error of the resulting probability being therefore in this case only 6^{-6}. The advantage of the method

is that the quantity which has here been called the argument of probability, depends in general more simply than does the probability itself on the conditions of a question; while the introduction of this new conception and nomenclature allows some of the most important known results respecting the mean results of many observations to be enunciated in a simple and elegant manner.

DONATIONS.

Historias e Memorias da Academia Real des Sciencias de Lisboa. Tome XII. Parte 2.

Discurso lido em 22 de Janeiro de 1843 *na sessao publica da Academia Real des Sciencias de Lisboa.* Por J. J. da Costa de Macedo. Presented by the Academy.

Le Petit Agriculteur. Par N. C. Seringe. Presented by the Author.

Astronomical Observations made at the Radcliffe Observatory, Oxford, in 1840. By M. J. Johnson, Esq. Presented by the Governors.

Archives du Museum d'Histoire Naturelle. Tome III. Liv. 3, et Tome II. Liv. 4. Presented by the Museum.

Remarks on Safety Lamps. By Doctor Reid Clanny, H. M. R. I. A. Presented by the Author.

Transactions of the Royal Society of Edinburgh. Vol. XV. Part 3. Presented by the Society.

Proceedings of the National Institution for the Promotion of Science at Washington. D. C. for 1840 *and* 1842. Parts 1 and 2. Presented by Thomas Sewall, M. D., Professor of Medicine in Columbia College, U. S.

Memoirs of the Chemical Society of London for 1841–3. Vol. I. Presented by the Society.

Numismatic Chronicle. No. XXII. Presented by the Numismatic Society.

Statistical Returns of the Dublin Metropolitan Police for 1842. Presented by the Commissioners.